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Solutions for Causal Modeling and A Technique for Measuring Causal, Broadband Dielectric Properties

Chad Morgan, Tyco Electronics
chad.morgan@tycoelectronics.com, 717-986-3342

Abstract

This paper addresses specific dielectric and conductor material relationships necessary to produce consistently causal and accurate electrical component models. The need for causal component models is highlighted by showing the types of digital waveforms that can result from simulating structures non-causally. A brief discussion of causal relationships that must be maintained between dielectric conductance and capacitance as well as conductor resistance and inductance is given. Ultimately, a measurement technique is outlined for producing causal permittivity data that is accurate across a broad frequency band. Final model data is provided in order to verify the accuracy and causality of the measurement method.

Author Biography

Chad Morgan earned his degree in Electrical Engineering from the Pennsylvania State University, University Park, in 1995. For the past 12 years, he has worked in the Circuits & Design group of Tyco Electronics as a signal integrity engineer, specializing in the analysis & design of high-speed, high-density components. Currently, he is a Member of Technical Staff with Tyco Electronics, where he focuses on high-frequency measurement & characterization of components & materials, full-wave electromagnetic modeling of high-speed interconnects, and the simulation of digital systems. Mr. Morgan's responsibilities at Tyco Electronics include numerous research activities, and he has presented multiple papers at both DesignCon and the International Microwave Symposium. Recent publications include:

“The Need for Impulse Response Models and an Accurate Method for Impulse Generation from Band-Limited S-Parameters”, DesignCon08, Track 7-WA1.

“An Accurate Technique for Measuring Broadband, Causal Electrical Properties of Dielectrics”, International Microwave Symposium 2007, Track WMC.

“Transition to Surface-Mount: An Analysis of Signal Integrity Improvement versus Manufacturing Concerns in Multi-Gigabit Systems Using High-Density Connectors”, DesignCon05 TecForum.

“Obtaining Accurate Device-Only S-Parameter Data to 15-20 GHz Using In-Fixture Measurement Techniques”, DesignCon04, Track 7-TP1.

“The Impact of PWB Construction on High-Speed Signals”, DesignCon99, Track H412.

Introduction

It is well-understood by today's signal integrity (SI) engineers that efficient design and verification of high-speed component electrical performance is best done through accurate modeling. When working in the multi-GHz frequency range, this modeling typically requires full-wave electromagnetic analysis. Many engineers work diligently to learn complicated 3D modeling tools so that they can accurately represent physical geometries and electrical boundary conditions. However, proper attention is not always given to the electrical parameters of the dielectrics and conductors used in the models. These material relationships, which must be causal to achieve causal model results, are often overlooked. This paper addresses the specific dielectric and conductor material relationships that are necessary for producing consistently causal and accurate models of broadband, digital components.

To begin, the paper highlights the need for causal component models by showing the types of digital waveforms that can result from simulating non-causal, partially causal, and completely causal structures. Once the importance of using causal models is shown, the paper then presents the underlying theory of causality, specifically in regards to the Hilbert transform and Kramers-Kronig relations. The paper shows how the Hilbert transform applies to the telegrapher's equations and how Kramers-Kronig relations apply to the complex permittivity of dielectrics.

The paper then focuses on the relationship that must be maintained between dielectric conductance and capacitance in order to achieve causal simulation results. Because these parameters are directly related to the dielectric's complex permittivity, attention is given to the relationship that must be kept between the real and imaginary portions of permittivity in order to guarantee causality. Applying causality principles to complex permittivity, a technique is then described for accurately measuring the *causal*, broadband permittivity of any non-permeable dielectric material (as may be common in typical printed circuit boards or connectors). The paper describes the measurement technique in detail and proves the validity of the method by comparing full-wave model data (using the derived permittivity) to measured component data.

Following the detailed discussion of causal dielectric relationships, the paper then briefly discusses causal conductor relationships. Specifically, a short summary is given concerning the relationship that must be maintained between resistance and inductance to achieve causal simulation results. Typically, this involves a causal surface impedance function.

The Causality Problem

In the past, passive component models could be distributed in lumped element format. One of the primary advantages to this method was the guaranteed passivity and causality of such models. However, modern digital signals now have edge rates that test the usability and accuracy of lumped component models. Therefore, today's passive component models are typically provided as distributed frequency-domain values, namely S-parameters. These frequency-domain models can quantify all linear electromagnetic effects and are very accurate.

As many SI engineers have seen, causality problems can result from converting seemingly accurate frequency-domain models into time-domain responses that clearly show output data either before absolute time zero or before some known 'base delay' of the component being analyzed.

In actuality, the observed non-causality is a manifestation of overall wave shape distortion in the time-domain. This time-domain wave shape distortion occurs when frequency-domain models are created without enforcing strict physical causality rules. In the upcoming section, examples of wave shape distortion and non-causality are shown, but one must understand the concept of ‘base delay’ causality before examining these waveforms.

‘Base Delay’ Causality

Typically, a time-domain waveform is considered causal if all of its data comes after absolute time zero. However, many passive component models can be causal and still show significant wave shape distortion due to improper frequency-domain modeling. Therefore, the concept of ‘base delay’ causality is introduced here.

In essence, one can evaluate a component model for proper wave shape much more stringently if one examines ‘base delay’ causality over absolute time zero causality. To do this, one must be able to determine the ‘base delay’ of a component, which is the first positive time point at which one would expect time-domain output data to occur.

There are multiple methods to get the ‘base delay’ of a specific frequency response for a component model. If a component has no reflections, the precise delay versus frequency can be calculated directly using equation (1). If a component has reflections, however, then the delay from equation (1) will show distortion ripples above and below the true delay of the component. One can still estimate the delay vs. frequency for this case by finding the center of the ripples.

$$Delay(f) = (Time \text{ for } 1 \lambda) \cdot (\# \text{ of } \lambda s) = \left(\frac{1}{f} \right) \cdot \left(\frac{-\text{unwrap}(\text{phase}([S]))}{360} \right) \quad (1)$$

The ‘base delay’ of the component is then the minimum delay at any frequency. It stands to reason that no time-domain data should occur before the ‘base delay’ of the component if no individual sinusoid can travel faster than that ‘base delay’. Figure 1 shows the delay versus frequency for a reflectionless insertion loss phase, as calculated by equation (1).

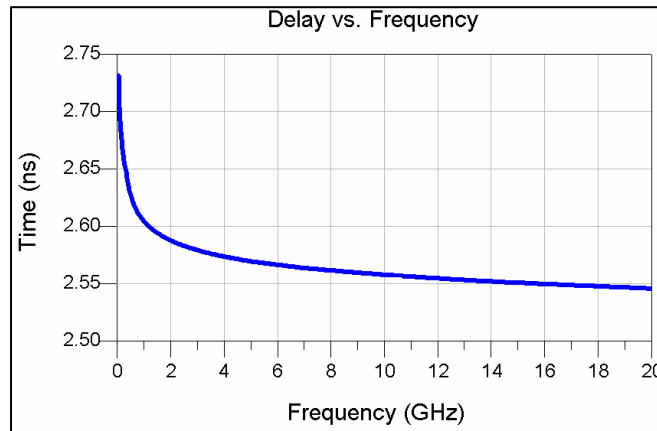


Figure 1: Delay vs. Frequency for Reflectionless Structure (Base Delay = 2.54 ns)

As stated previously, the calculation of ‘base delay’ is important because it provides an obvious method for checking ‘base delay’ causality, which in turn provides detection of not-so-obvious wave shape distortion.

Non-Causal Wave Shapes

To understand ‘base delay’ causality and how it highlights overall wave shape distortion, it is helpful to examine several time-domain waveforms for a specific component. Consider the case of a reflectionless 50-Ohm stripline structure ($\sim 15''$).

First, equation (1) is used to calculate the delay versus frequency of the component. This delay is shown in Figure 2 when the component is modeled in several different ways. Regardless of the modeling method, the ‘base delay’ of the component is clearly 2.54 ns. This ‘base delay’ value is then helpful in examining the validity of various impulse responses for the component, also shown in Figure 2.

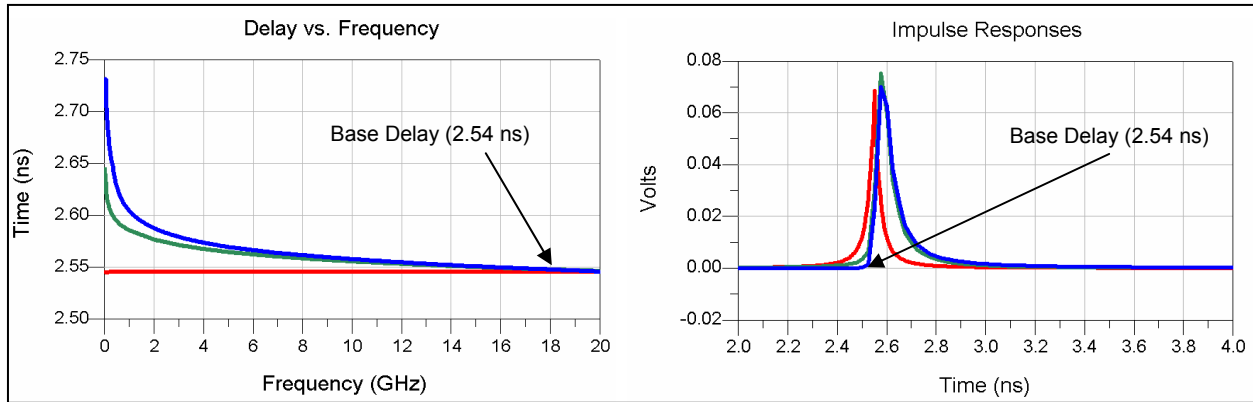


Figure 2: Delay vs. Frequency & Impulse Responses (Blue=Causal, Green=Partially Causal, Red=Non-Causal)

Upon examination of the impulse responses in Figure 2, it should be clear that the blue response is ‘base delay’ causal, and therefore, physically valid. The red and green waveforms, however, violate ‘base delay’ causality and are invalid. For this example, the blue waveform comes from a model using causal dielectric and conductor properties. The green waveform, however, comes from a model using causal dielectric properties but non-causal conductor properties. The red waveform uses non-causal properties for both the dielectrics and conductors.

To further illustrate how ‘base delay’ causality can be used to detect wave shape distortion, Figure 3 shows the step and pulse responses for the stripline example. Once again, it is apparent that the blue waveforms are physically valid while the red and green waveforms are not.

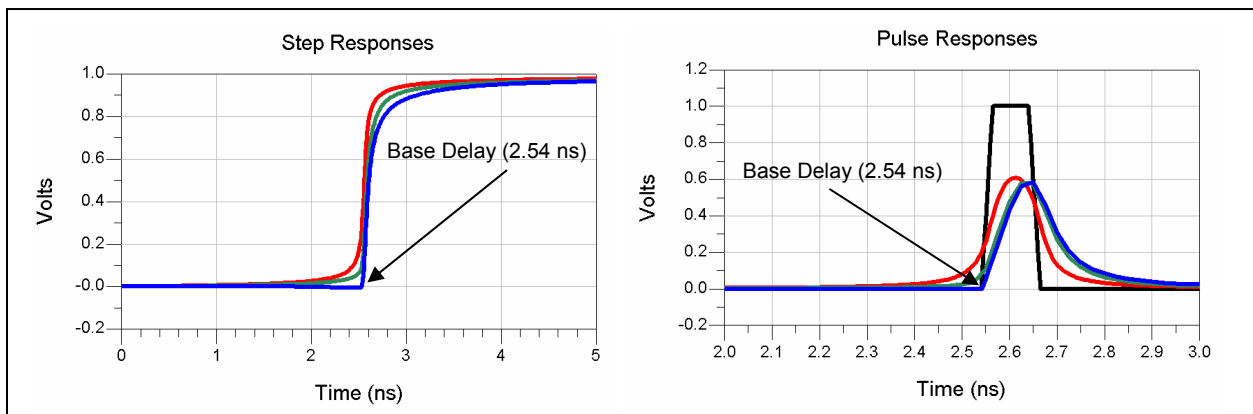


Figure 3: Step Responses & Pulse Responses (Blue=Causal, Green=Partially Causal, Red=Non-Causal)

Yet another example of how ‘base delay’ causality can be used to detect wave shape distortion is seen in Figure 4, where several PRBS 2^7 eye patterns running at 10 Gbps are displayed. Note that the black waveforms in Figure 4 represent the component’s ‘base delay’ (i.e. It is the output eye pattern of a lossless, dispersionless component that is 2.54 ns long).

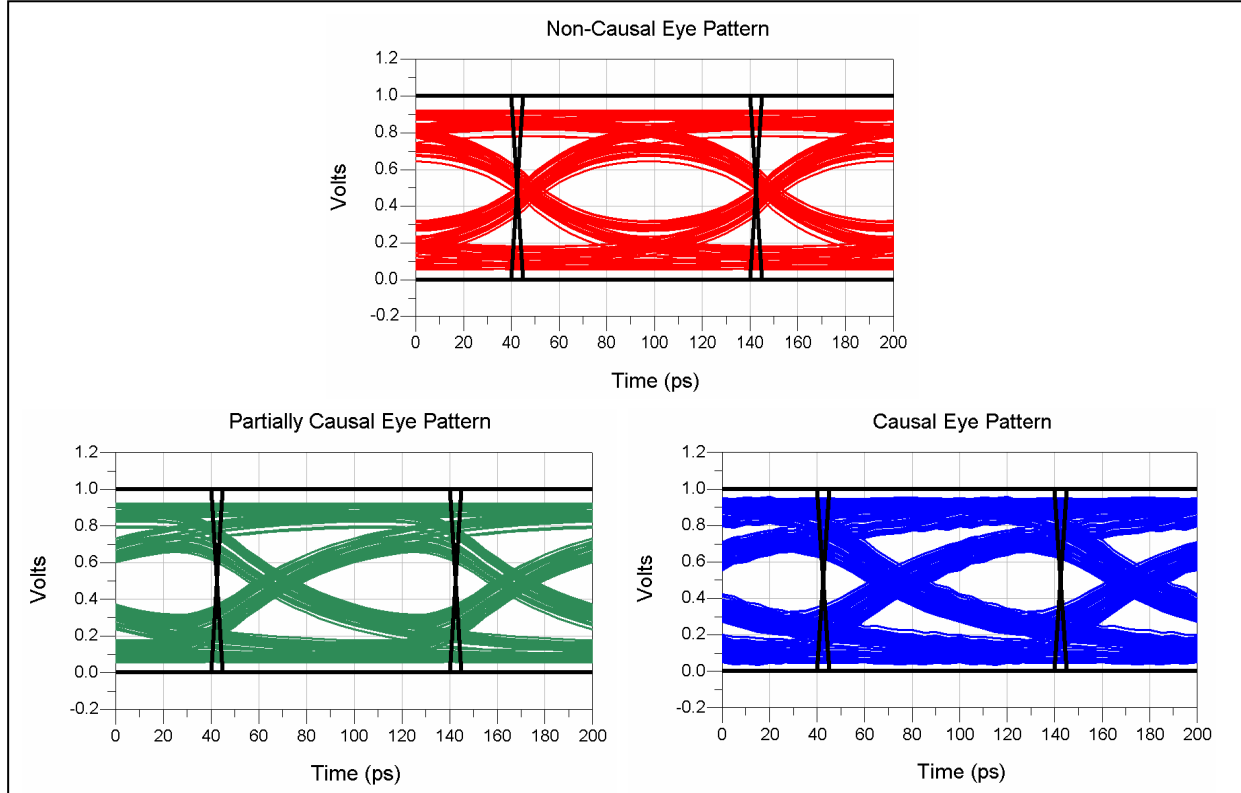


Figure 4: Eye Patterns (Blue=Causal, Green=Partially Causal, Red=Non-Causal)

In the case of the red waveform in Figure 4, it is obvious that the model is invalid, because the eye pattern does not display the expected ‘shark fin’ wave shape. However, the invalidity of the green waveform in Figure 4 is not as obvious until one examines the starting point of a rising edge in comparison to the starting point of a rising edge for the baseline black waveform. The blue waveform in Figure 4 is the most causal, and therefore the most physically valid.

Ultimately, one must be sure that all component models are ‘base delay’ causal in the frequency domain in order to ensure proper wave shape in the time domain. Improper wave shape during time-domain simulation is highly undesirable, as it can cause errors during jitter analysis, eye pattern mask compliance evaluation, and digital equalization tap optimization.

‘Base delay’ causality can only be achieved when one enforces strict causality constraints on both the dielectric and conductor properties of the physical model. Methods for enforcing these constraints will be addressed throughout the paper. Interestingly, although enforcing both dielectric and conductor causality is important, Figures 2-4 indicate that dielectric causality may be more important than conductor causality, since dielectric causality violations tend to produce larger wave shape distortions in the example provided.

Causality Fundamentals

‘Base delay’ causality, and therefore proper wave shape, can only be achieved for a component model by enforcing strict dielectric and conductor causality constraints. These constraints will be discussed in this section. Before doing so, however, a review of the Hilbert transform and Kramers-Kronig (K-K) relations is in order.

Transform Summary

For any real-valued, causal time-domain function, the real and imaginary portions of its spectrum are related by the continuous Hilbert transform, shown in equation (2). For simplicity, only the real-to-imaginary version of the Hilbert transform is included, but there are other versions of the Hilbert transform, including the imaginary-to-real conversion and also relationships between magnitude and minimum phase.

$$X_I(e^{j\omega}) = -\frac{1}{2\pi} P \int_{-\pi}^{\pi} X_R(e^{j\theta}) \cot\left(\frac{\omega - \theta}{2}\right) d\theta \quad (2)$$

Note the difficulties with the continuous Hilbert transform, namely the Cauchy principle value (P) included in the integration. Because measured and modeled frequency-domain data is almost always discrete in nature, these difficulties can be eliminated by considering the discrete Hilbert transform, as shown in equation (3).

$$\tilde{X}_I[k] = \frac{1}{N} \sum_{m=0}^{N-1} \tilde{X}_R[m] \cdot \tilde{V}_N[k-m] \quad \tilde{V}_N[k] = \begin{cases} -j2 \cot(\pi k/N), & k \text{ even} \\ 0, & k \text{ odd} \end{cases} \quad (3)$$

The discrete Hilbert transform is similar to the discrete Fourier transform (DFT). As a result, one can take advantage of pre-written DFT (and FFT) algorithms to compute the discrete Hilbert transform, as shown in Figure 5.

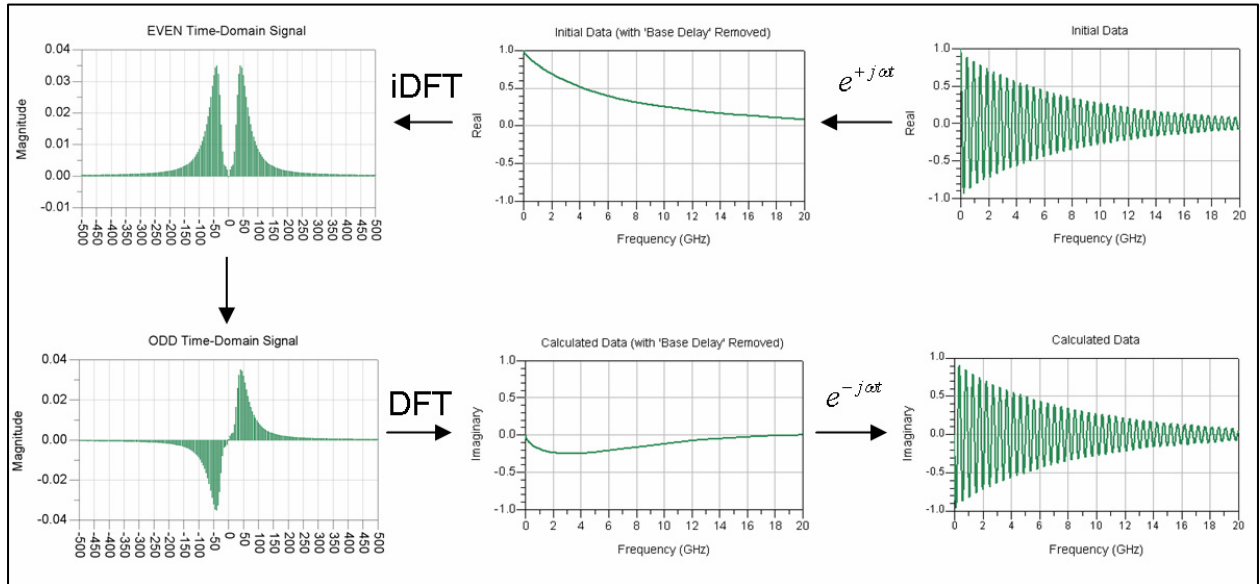


Figure 5: ‘Base Delay’ Causal Hilbert Transform Method

What Figure 5 demonstrates is that one can use only the real portion of any frequency-domain function to directly calculate the imaginary portion of the function that will guarantee a causal time-domain result. The specific example of Figure 5 shows an S-parameter's imaginary spectrum being calculated from only its real spectrum and the 'base delay' value (t). The final spectrum then guarantees a 'base delay' causal result.

It is important to recognize that the continuous and discrete Hilbert transforms can be applied to values other than S-parameters. For example, causal relationships, as dictated by the Hilbert transform, are discussed in this paper between dielectric capacitance and conductance, conductor resistance and inductance, and real and imaginary permittivity. In the case of complex permittivity, the imaginary portion (ϵ'') can be calculated from the real portion (ϵ') as shown in equation (4), which is one of the K-K relations.

$$\epsilon''(\omega) = \frac{2\omega}{\pi} \cdot P \int_0^{\infty} \frac{\epsilon'(\Omega) - \epsilon_0}{\Omega^2 - \omega^2} d\Omega \quad (4)$$

Materials Relationships

In this paper, the primary point of interest is to determine the dielectric and conductor frequency-domain relationships that must be enforced during component modeling to guarantee causal time-domain results from the final component model.

To help understand these dielectric and conductor relationships, consider the case of a matched-impedance, uniform transmission line operating under quasi-TEM conditions. In that case, the performance of the transmission line can be described by the propagation constant, γ , which consists of the telegrapher's values for resistance (R), inductance (L), conductance (G), and capacitance (C) versus frequency. This is shown in equation (5).

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (5)$$

Examination of equation (5) reveals that the conductor terms ($R + j\omega L$) and the dielectric terms ($G + j\omega C$) are independent of one another in regards to causality. To understand this, remember that multiplication in the frequency domain is equivalent to convolution in the time domain. Because ($R + j\omega L$) and ($G + j\omega C$) are independent multiplication terms in the frequency domain, one can also think of them as being independent convolution terms in the time domain. As a result, if either term is non-causal, then the convolution of the two terms (frequency-domain product) will also be non-causal. The final implication is that the conductor and dielectric terms are independent, and both terms must be causal for the final response to be causal.

Once the independence of the dielectric and conductor terms in equation (5) is established, the causality constraints for each term become clear. Each term is a complex frequency-domain function, and as stated earlier, the real and imaginary parts of such functions must be related by the Hilbert transform in order to be causal. Therefore, the final response of equation (5) will only be causal if the conductor terms (R and ωL) are related by the Hilbert transform and if the dielectric terms (G and ωC) are related by the Hilbert transform. These relationships are shown graphically in Figure 6.

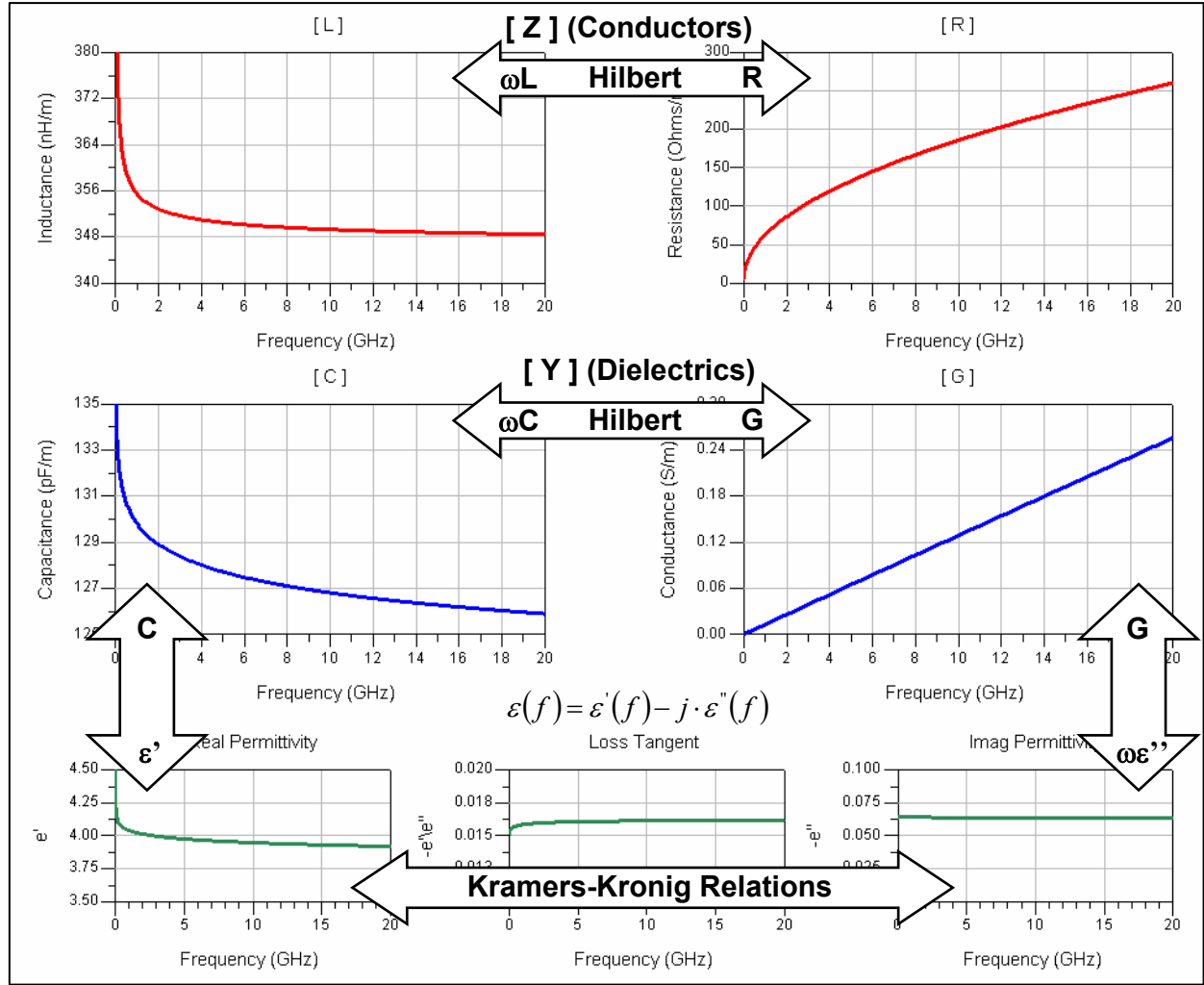


Figure 6: Causality Relationships for RLGC Telegrapher's Parameters

One of the easiest ways to be sure that conductor and dielectric terms satisfy the constraints shown in Figure 6 is to use pre-defined, causal *functions* during modeling. For conductor terms, this is typically done by using a resistance term that varies with the square root of frequency and an inductance term that varies with the inverse of the square root of frequency. This relationship is shown in equation (6).

$$R(f) \propto \sqrt{f} \longleftrightarrow L(f) \propto 1/\sqrt{f} \quad (6)$$

For dielectric terms, one can also use pre-defined, causal *functions* during modeling. However, such functions are typically applied to complex permittivity, rather than capacitance and conductance directly. Using causal permittivity *functions* automatically enforces G-to- ω C causality, due to the physical relationships defined by equation (7).

$$C(f) \propto \varepsilon'(f) \longleftrightarrow G(f) \propto \varepsilon''(f) \cdot 2\pi f \quad (7)$$

(Combining equation (7) relations with the G-to- ω C Hilbert relationship shown by Figure 6 quickly shows that ε' and ε'' must also be related by the Hilbert transform – i.e. K-K relations).

Enforcing Dielectric Causality

This paper demonstrates a method for obtaining a causal dielectric function that inherently satisfies the Hilbert transform relationship between its real and imaginary permittivity. As a review, equation (8) shows the various ways that complex permittivity can be expressed using real permittivity (ϵ'), imaginary permittivity (ϵ''), loss tangent ($\tan\delta$), or conductivity (σ).

$$\epsilon = \epsilon' - j\epsilon'' = \epsilon'(1 - j \tan \delta), \text{ where } \tan \delta = \frac{\epsilon''}{\epsilon'}, = \frac{\sigma}{\omega\epsilon'} \quad (8)$$

Besides being causal, it is important that the final function also be accurate across a broad frequency band. Ultimately, two items are necessary to successfully derive accurate and causal permittivity. The first is an accurate measurement method for examining a material's broadband permittivity. The second is a causal function that can be fit to the measured data.

Measurement Method for Complex Permittivity

The following is a description of a 'throughput' technique for measuring the broadband complex permittivity for any non-permeable, homogeneous dielectric. Because digital component models require broadband permittivity values, resonant techniques are not preferred, as they typically provide data at only one frequency. Instead, a 'throughput' technique is described that makes use of homogeneous dielectric stripline structures.

To begin, homogeneous, uniform, 50-Ohm stripline structures need to be constructed with known cross sectional dimensions and a smooth metal with known conductivity (typically copper at $5.8e7$ S/m). Several examples of such stripline structures are shown in Figure 7, including two structures in printed circuit board FR4 and one structure in typical connector plastic.

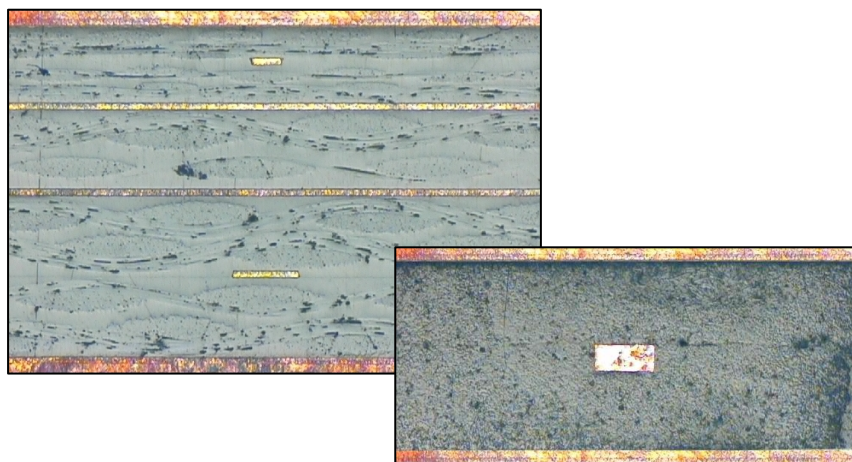


Figure 7: Printed Circuit Board and Connector Plastic Stripline Structures

Note that each stripline sample must be constructed in several carefully chosen lengths in order to complete TRL calibration ^{[1][2]} during the vector network analyzer (VNA) measurement process. These varying stripline lengths serve to eliminate TRL calculation singularities. Once stripline structures have been built, TRL calibration is completed with the VNA in order to remove unwanted cable and test point fixturing effects. The final measurement then consists only of the response from a reflectionless, uniform, homogeneous stripline structure of some known length. Figure 8 shows the measurement setup with several example fixtures.

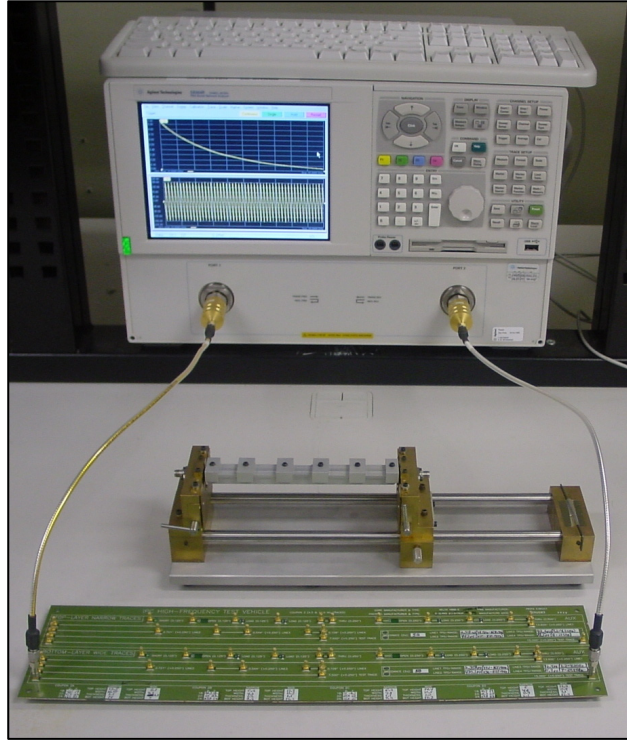


Figure 8: VNA Measurement Setup with Fixturing

Upon completion of TRL calibration and measurement of the reflectionless, known-length stripline structure, one should observe insertion and return loss data similar to that shown in Figure 9. In linear format, the insertion loss should roll off exponentially, and the return loss should stay below 10% to avoid error greater than 1% in the insertion loss. In dB format, the insertion loss should be linear, and return loss should stay below approximately -20 dB.

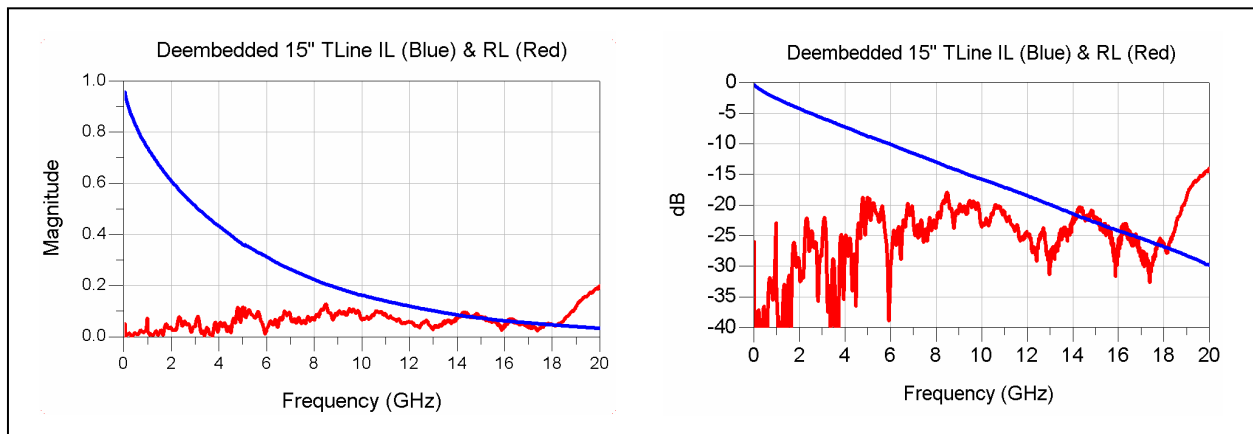


Figure 9: Measured Stripline Data Following Test Point Deembedding

It is important that reflection resonances are not observed on insertion loss or insertion phase, because these values are then used to calculate the measured dielectric constant (ϵ') and loss tangent ($\tan\delta$). Smooth insertion loss and insertion phase are an indication of a successful TRL calibration and a good measurement.

Post-Processing Measurement Results

Once the stripline's insertion loss and insertion phase have been measured, the material's dielectric constant and loss tangent can be derived. To calculate the dielectric constant, one must first calculate delay versus frequency, as described by equation (1). Given this delay versus frequency and the length of the stripline, equation (9) can then be used to directly calculate the material's dielectric constant versus frequency.

$$v(f) = \frac{c_0}{\sqrt{\epsilon'(f)}} = \frac{Length}{Delay(f)} \longrightarrow \epsilon'(f) = \left(\frac{c_0 \cdot Delay(f)}{Length} \right)^2 \quad (9)$$

Note that the dielectric constant calculated by equation (9) is not the true dielectric constant of the material. In fact, this dielectric constant is artificially high due to parasitic inductance from the stripline's signal conductor. As Figure 10 shows, parasitic inductance error grows at lower frequencies, because more fields can penetrate the signal conductor at those frequencies. For the time being, the calculated permittivity will be used as an estimate, and a method will be proposed later for removing the artificial inductive delay.

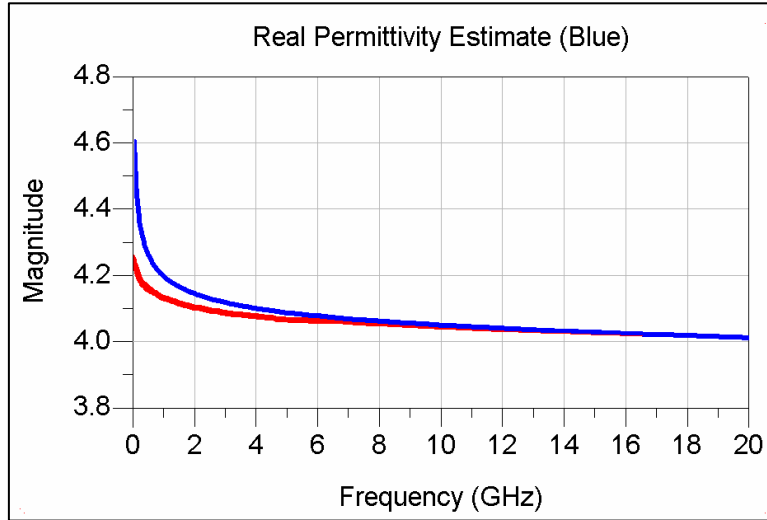


Figure 10: Calculated Permittivity Estimate (Blue) vs. Actual Permittivity (Red)

To calculate the dielectric's loss tangent, equation (10) is used^[3]. Given the previously estimated permittivity, the only unknown left in equation (10) is the conductor loss ($\alpha_{conductor}$).

$$\tan \delta(f) = \frac{\alpha_{dielectric} \cdot c_0}{\pi \cdot \sqrt{\epsilon'(f)} \cdot f}, \text{ where } \alpha_{dielectric} = \alpha_{total} - \alpha_{conductor} = \frac{-\ln|IL|}{l} - \alpha_{conductor} \quad (10)$$

To determine the stripline's conductor loss, and ultimately the dielectric's loss tangent, the procedure shown in Figure 11 is used. In this procedure, the calculated permittivity and the dimensions of the stripline are used to calculate the loss of the structure when loss tangent is zero. The result is conductor loss alone, which can then be subtracted from the measured total loss to yield dielectric loss. Dielectric loss can then be used to calculate the loss tangent of the stripline's dielectric material. Note that, as with permittivity, the calculated loss tangent is still an estimate, since its calculation relies on an estimated permittivity value. A method will be outlined next for removing residual error from the measured dielectric constant and loss tangent.

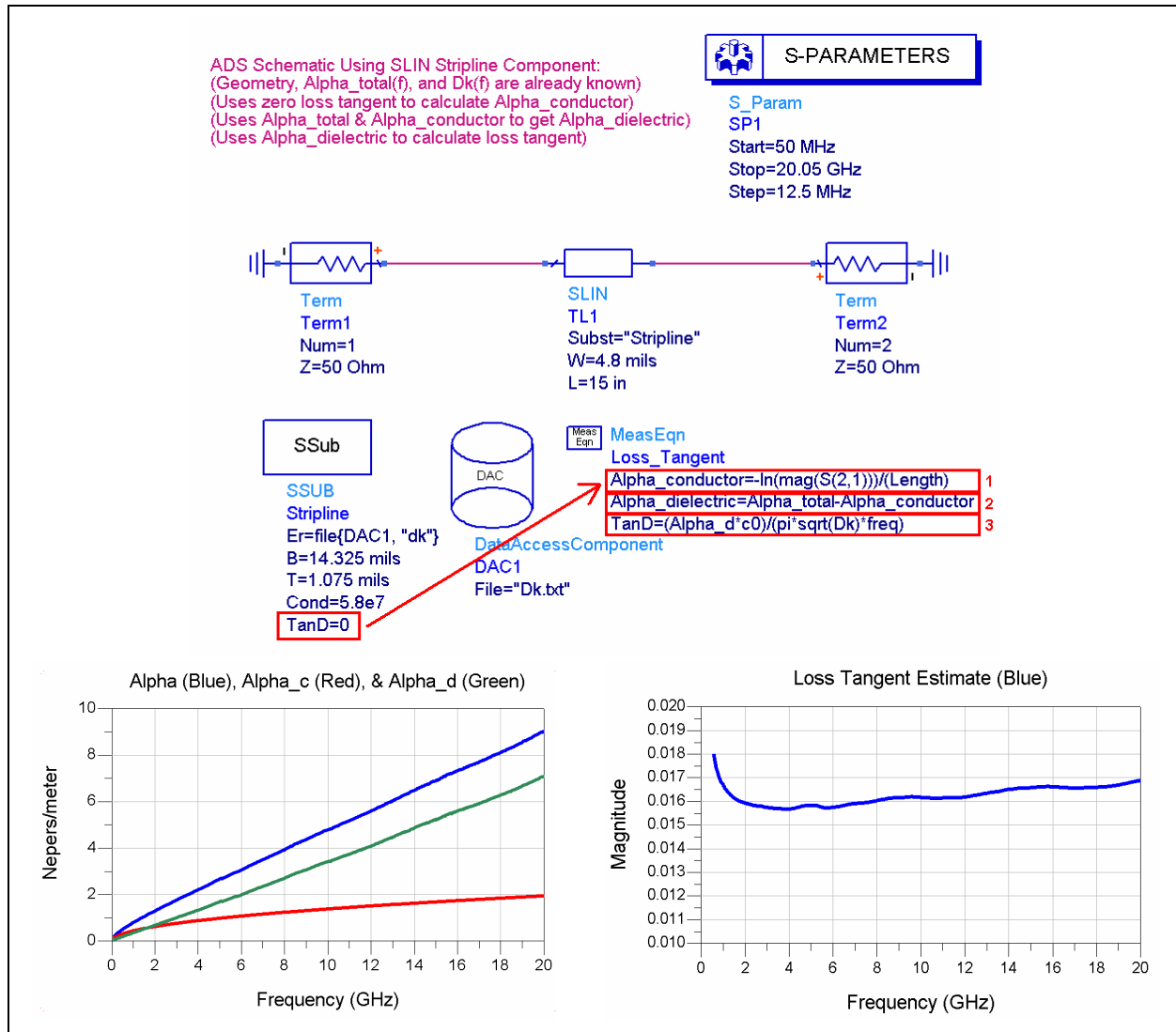


Figure 11: Separating Conductor and Dielectric Losses to Calculate Loss Tangent versus Frequency

Physics-Based Causal Functions

Removal of residual dielectric constant and loss tangent error and enforcement of causality involves the selection of a physics-based causal function and the fitting of that function to the estimated dielectric constant and loss tangent data.

To find an appropriate causal function for complex permittivity, one must first understand how dielectric materials behave in different frequency bands. Figure 12 shows that dielectric behavior is dominated by ionic and dipolar relaxation mechanisms from DC to several hundred GHz, which is the frequency band of interest for modern digital systems.

In the ionic and dipolar relaxation frequencies, complex permittivity can be described, for individual molecules, by the Debye model. Figure 13 shows both the causal Debye function, along with plots of the real permittivity, imaginary permittivity, and loss tangent that are described by the Debye model.

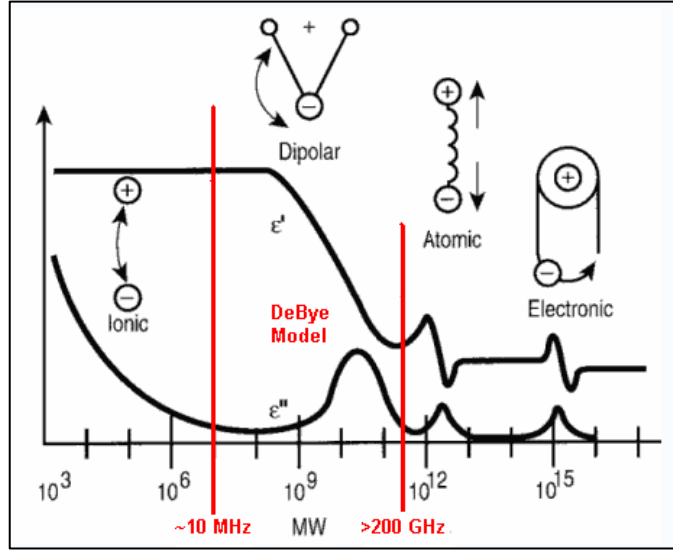


Figure 12: Molecular Complex Permittivity Behavior versus Frequency

It should be clear that the Debye model does well in describing the dipolar relaxation behavior of individual molecules. However, measurements of many common PCB and connector dielectrics have shown that the dielectric constant and loss tangent of these materials do *not* follow the Debye model. In fact, although many materials do show a monotonically dispersive dielectric constant versus frequency, they typically do *not* show the rising and falling loss tangent versus frequency of Figure 13. Rather, most materials show a fairly flat loss tangent versus frequency. The discrepancy between the Debye model and real material behavior can be reconciled by considering that real materials are made of many molecules; therefore, their complex permittivity behavior is actually the summation of many Debye functions, not a single Debye function.

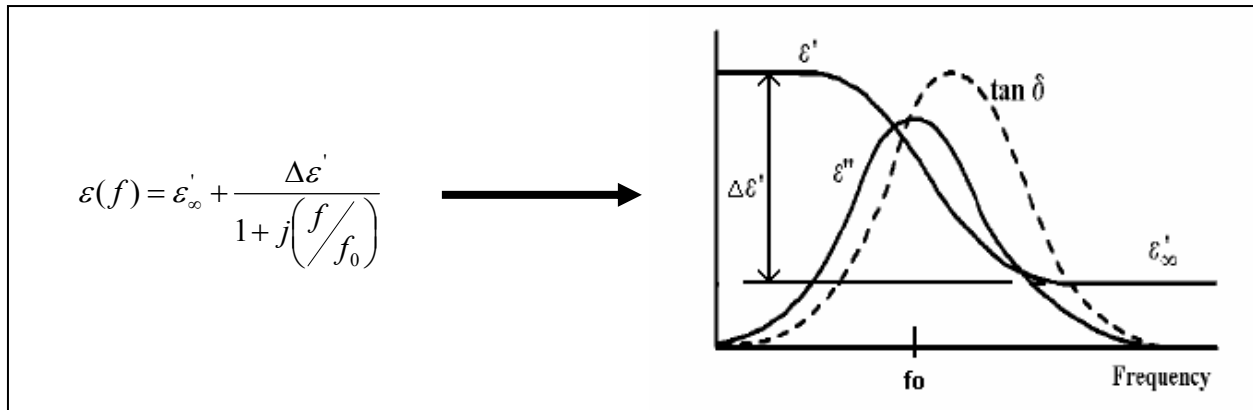


Figure 13: Debye Model Describing Dipolar Relaxation at Microwave Frequencies

Ultimately, another causal function, showing a dispersive dielectric constant and a flat loss tangent, must be fit to the measured data. Luckily, just such a function has been derived in recent papers ^{[4][5]} and is shown in Figure 14. In essence, this function (which will be referred to as the Σ Debye model) is the result of summing infinitely many Debye models and then simplifying that infinite summation into a finite calculation. Physically, the function represents the fact that real materials have a diverse molecular composition.

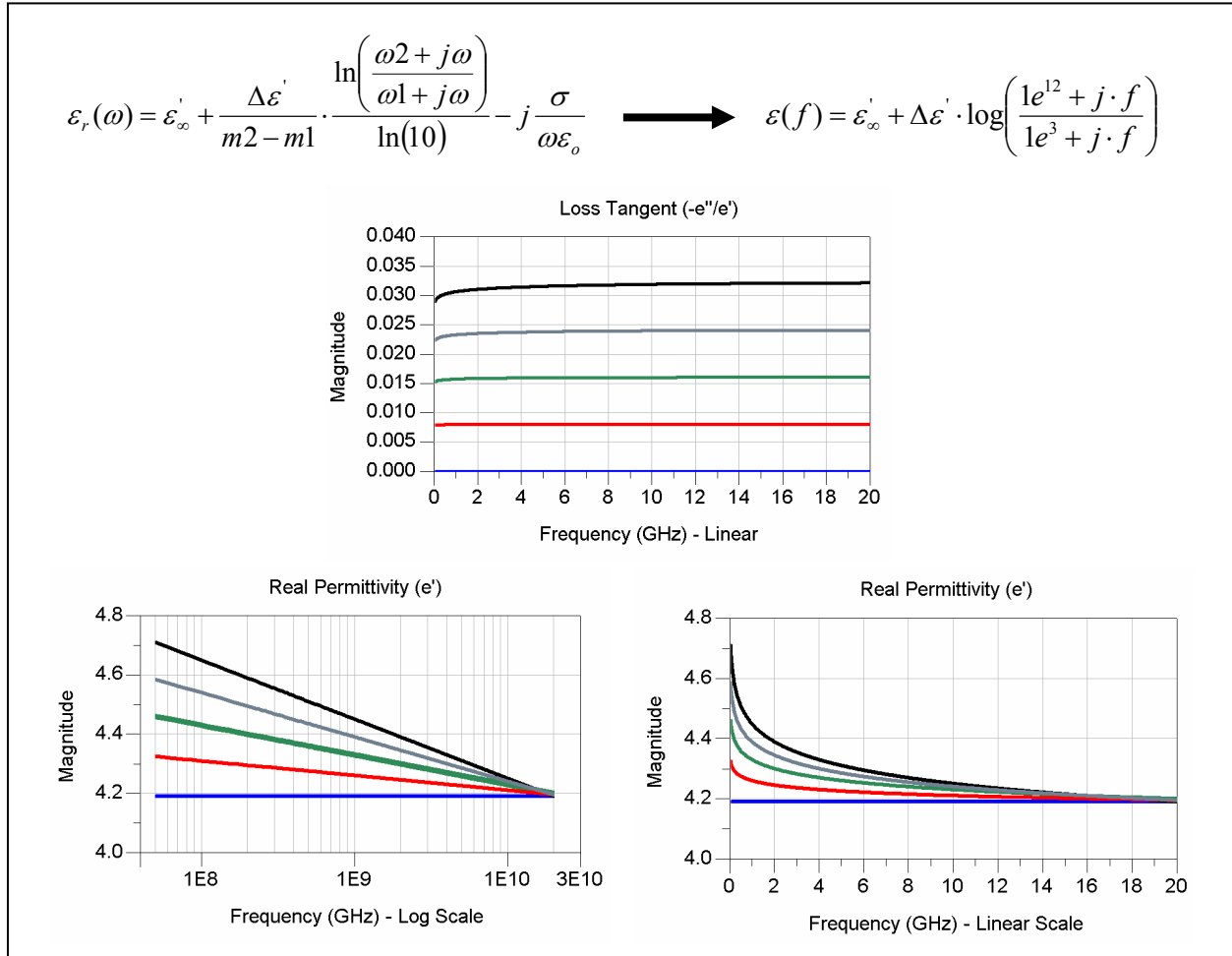


Figure 14: Σ Debye Model Simplification and Loss-Dispersion Relationships

Examining the upper left equation of Figure 14, it is first desirable to simplify the equation by removing the $\sigma/\omega\epsilon_0$ bulk conductivity term, since it is only important at frequencies below ~ 100 Hz. One can then complete other simplifications, such as replacing $\omega=2\pi f$ with frequency and replacing $\ln(x)/\ln(10)$ with $\log_{10}(x)$. Finally, one should set $f1$ to a lower frequency bound and $f2$ to an upper frequency bound that will make the function behave as desired for SI frequencies of interest. For this paper, $f1$ is set to 1 kHz and $f2$ is set to 1 THz.

The upper right equation of Figure 14 is the resulting simplified equation. Note that the equation is *inherently causal*. Also, only two terms remain that must be varied in order to fit the function to the previously measured permittivity data. These two terms are the real permittivity at high frequency (ϵ'_∞) and the log slope of the real permittivity ($\Delta\epsilon'$).

The simplified Σ Debye function in Figure 14 must be understood, as it will be curve fit to the measured permittivity data in order to remove parasitic errors and enforce causality. Note that this function is the result of physical material studies, so it should describe dielectrics well. Also, for materials that follow this model, the function mandates that *higher loss tangent be accompanied by more dielectric constant dispersion*. This phenomenon is shown by the three charts in Figure 14, which clearly demonstrate increased ϵ' dispersion with higher $\tan\delta$.

Curve-Fitting and Validation Data

At this stage of the measurement process, estimated dielectric constant (ϵ') and loss tangent ($\tan\delta$) have been determined, and a suitable function has been found that should be causal and accurate. The next step is to curve-fit the Σ Debye model to the estimated permittivity data in order to remove parasitic inductance error and enforce causality.



Figure 15: Printed Circuit Board Dielectric Validation Example – Nelco 4000-6 FR4

Figure 15 shows a Nelco 4000-6 FR4 printed circuit board that is used as an example for demonstrating the curve-fitting process. The previously estimated dielectric constant and loss tangent are shown by the blue waveforms of Figure 16. The two variables, ϵ'_∞ and $\Delta\epsilon'$, of the Σ Debye function are then tuned until a suitable curve fit to the estimated data is obtained. The final function data, with $\epsilon'_\infty = 3.76$ and $\Delta\epsilon' = 0.094$, is shown by the red waveforms of Figure 16.

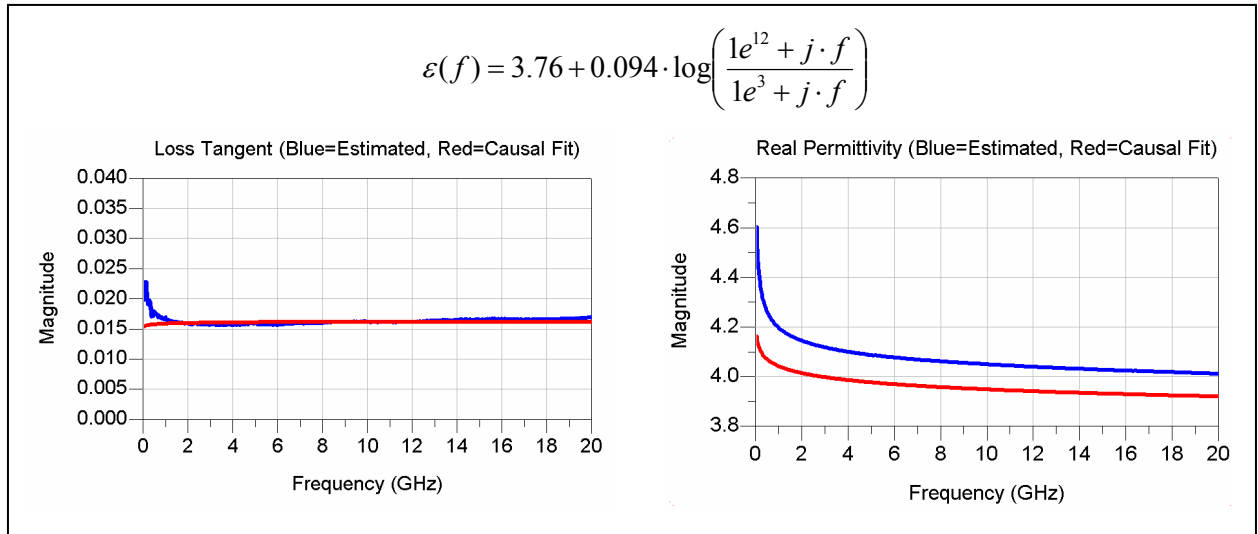


Figure 16: PCB Dielectric Curve Fit Data (Blue=Measured Data, Red=Curve Fit)

As expected, the Σ Debye model variables are tuned until the function loss tangent matches the estimated loss tangent. However, in this case, the Σ Debye model variables must be tuned such that the function dielectric constant is ~ 0.1 lower than the estimated dielectric constant in order to obtain accurate model results. This ‘aim-low’ factor is determined by using the final complex permittivity data in a full-wave model of the stripline structure and comparing that model’s results to directly measured data. Although unexpected, the ‘aim-low’ factor of ~ 0.1 for this dielectric proves necessary and is reasonably simple to determine through modeling.

Figure 17 demonstrates the accuracy of the final Σ Debye permittivity data for the printed circuit board example. In Figure 17, the blue waveforms represent the measured responses of the stripline trace, while the red waveforms represent the modeled responses of the stripline trace when using the Σ Debye permittivity data.

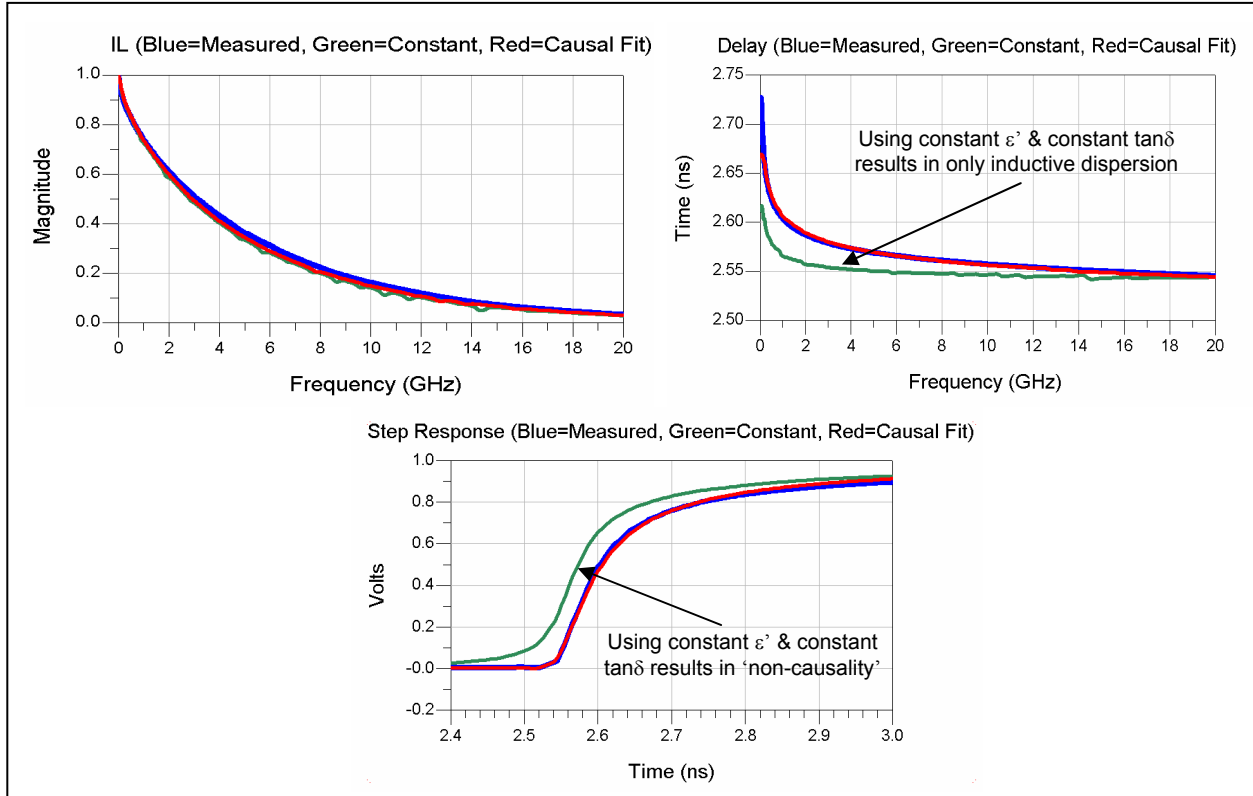


Figure 17: Test Data (Blue) & Model Data (Green=Constant ϵ' & $\tan\delta$, Red= Σ Debye ϵ' & $\tan\delta$) Correlation

When examining the correlation between the blue and red waveforms of Figure 17, two points are striking. First, it is clear that the Σ Debye model provides a complex permittivity that yields extremely accurate wave shapes in the frequency domain, both for insertion loss and delay versus frequency. Second, the Σ Debye model provides a permittivity that yields accurate and 'base delay' causal wave shapes in the time domain, as shown by the step responses.

Because the final Σ Debye permittivity data yields such accurate and causal model results during full-wave 3D modeling, it is fair to say that the permittivity data from the final Σ Debye function is the *actual* permittivity data for the material. At this time, it is not fully understood why the Σ Debye dielectric constant data must be fit lower than the measured dielectric constant data. In the future, it is desirable to explain the reasoning behind this, but for the time being, it is relatively simple to determine this 'aim-low' factor through several full-wave model iterations.

Note that the green waveforms in Figure 17 show modeled data when a single value for dielectric constant and a single value for loss tangent are used across all frequencies. This data is clearly inaccurate and non-causal, which highlights the fact that *accurate and causal time-domain model results cannot be achieved using constant ϵ' and $\tan\delta$ versus frequency*.

As another example for demonstrating the curve-fitting process, Figure 18 shows another stripline structure, this time using an unfilled PPO connector plastic.

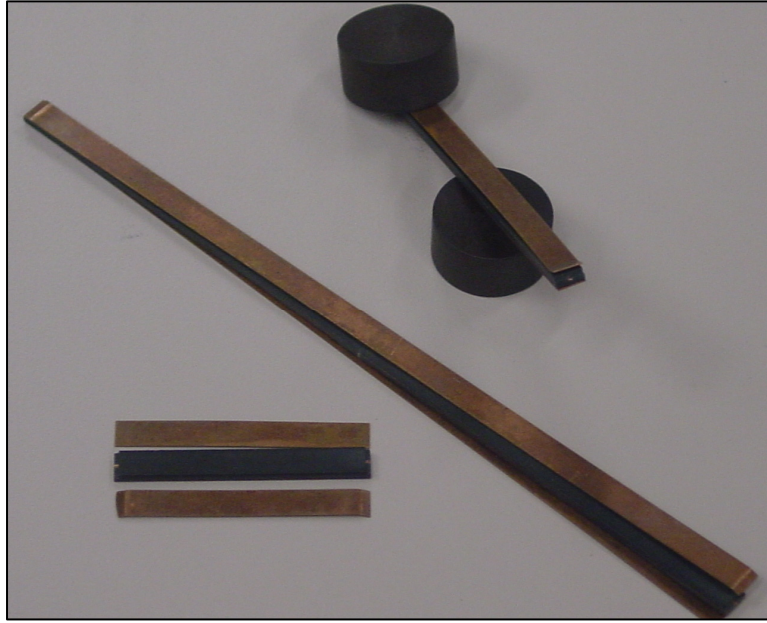


Figure 18: Connector Dielectric Validation Example – Unfilled PPO

For this structure, the estimated dielectric constant and loss tangent are shown by the blue waveforms of Figure 19. The two variables, ϵ'_{∞} and $\Delta\epsilon'$, of the Σ Debye function have been tuned to achieve a suitable curve fit to the estimated data. The final function data, with $\epsilon'_{\infty} = 2.703$ and $\Delta\epsilon' = 0.025$, is shown by the red waveforms of Figure 19.

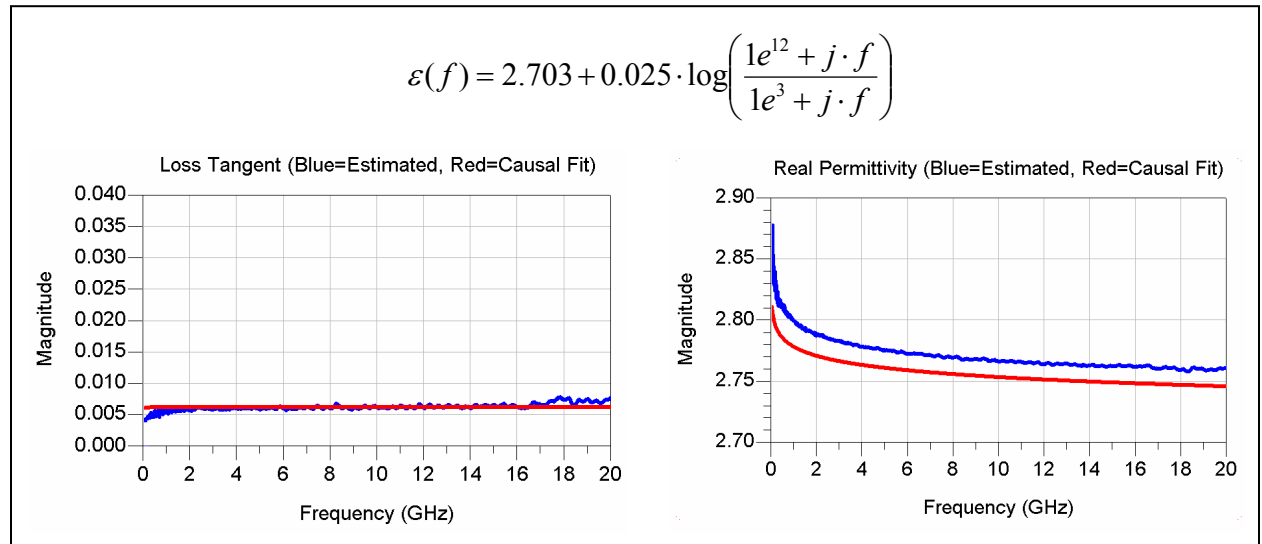


Figure 19: Connector Dielectric Curve Fit Data (Blue=Measured Data, Red=Curve Fit)

As in the previous example, the Σ Debye variables are tuned to make the measured and Σ Debye loss tangents agree. In this case, however, the Σ Debye model variables are tuned such that the Σ Debye dielectric constant is ~ 0.01 lower than the estimated dielectric constant. This ‘aim-low’ factor, though less than the previous example, proves necessary to obtain accurate model results.

Figure 20 demonstrates the accuracy of the final Σ Debye permittivity data for the connector dielectric example. In Figure 20, the blue waveforms represent the measured responses of the stripline trace, while the red waveforms represent the modeled responses of the stripline trace when using the Σ Debye permittivity data.

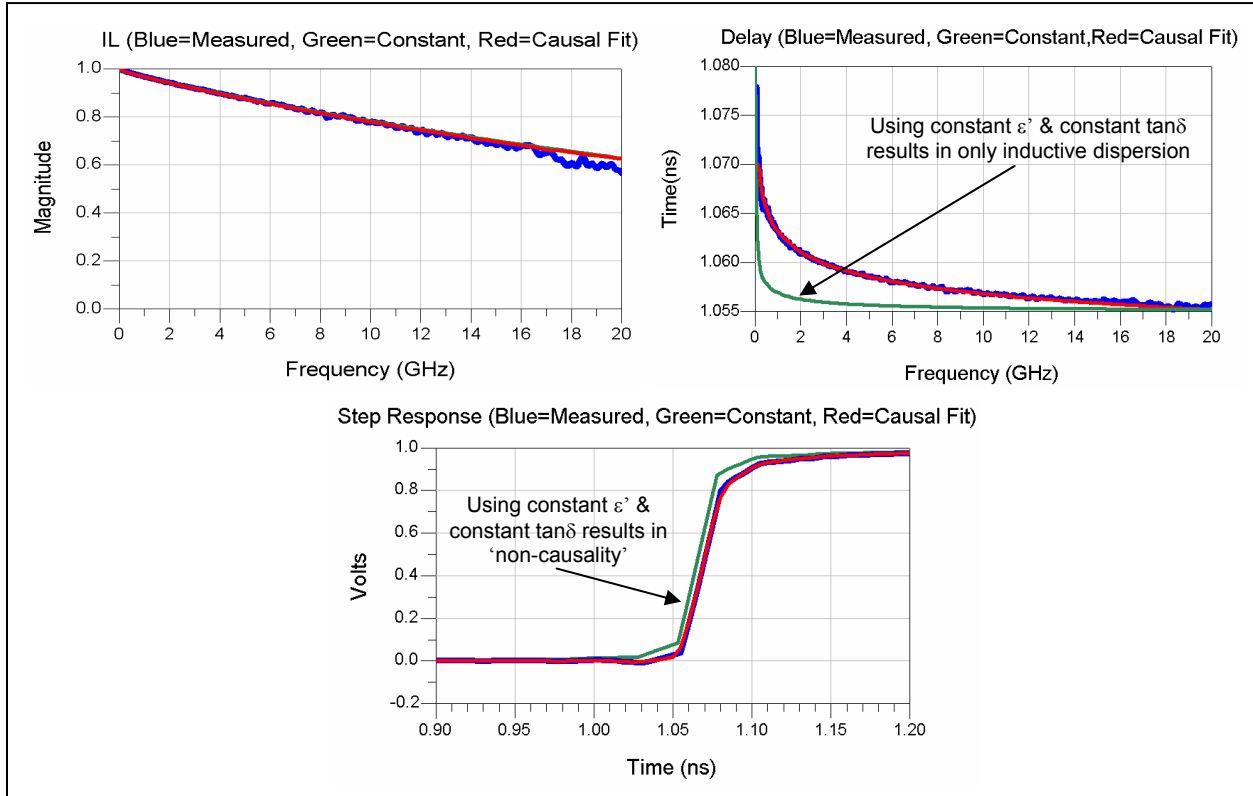


Figure 20: Test Data (Blue) & Model Data (Green= $\text{Constant } \epsilon' \text{ \& } \tan\delta$, Red= Σ Debye $\epsilon' \text{ \& } \tan\delta$) Correlation

As with the previous example, the Σ Debye model's complex permittivity yields extremely accurate insertion loss and delay versus frequency wave shapes. The time-domain step response is also 'base delay' causal, and the wave shape correlation is very good.

In order to achieve such excellent correlation, the 'aim-low' factor for this dielectric was approximately 0.01. Although not completely proven, it appears that the 'aim-low' factor necessary to achieve accurate model results is smaller for dielectrics with lower loss.

Figure 20 again shows green waveforms, which represent modeled data when a single value for dielectric constant and a single value for loss tangent are used across all frequencies. This data is clearly inaccurate and non-causal, which again highlights that fact that *accurate and causal time-domain model results cannot be achieved using constant ϵ' and $\tan\delta$ versus frequency.*

Ultimately, an accurate measurement technique and curve-fitting process utilizing the Σ Debye model has been demonstrated that produces extremely accurate model data which is guaranteed to be 'base delay' causal. One can export tabular dielectric constant and loss tangent data versus frequency for import into various modeling tools, but one should be cautious of frequency-domain interpolation of tabular data, as interpolation itself can lead to slightly non-causal results.

Enforcing Conductor Causality

As described earlier in the paper, a physical component model's response will only be 'base delay' causal if *both* the dielectric and conductor properties are implemented in a causal fashion. Although initial waveform examination appears to show that enforcement of dielectric causality is more important than the enforcement of conductor causality, a discussion of conductor causality is still in order. In this section, a common function for conductor properties is discussed that is guaranteed to produce causal and reasonably accurate results.

Earlier in the paper, equation (6) listed some specific conductor properties that must be maintained to achieve causal conductor responses, namely that resistance should be proportional to the square root of frequency and inductance should be proportional to the inverse of the square root of frequency. To be more specific, equations (11) and (12) list exact functions that are commonly used to estimate resistance and inductance. The use of these two functions in combination guarantees causal conductor responses.

$$R(f) = R_{dc} + R_s \sqrt{f} \quad (11)$$

$$L(f) = L_{ext} + \frac{R_s}{2\pi} \left(\frac{1}{\sqrt{f}} \right) \quad (12)$$

In equation (11), R_{dc} represents the low frequency resistance of a structure where fields can penetrate the entire cross section of the signal conductor(s). R_s , on the other hand, is simply a constant that is determined by curve fitting the function to modeled resistance versus frequency. In equation (12), L_{ext} is the external (or high frequency) inductance of a structure when no fields can penetrate the cross section of the signal conductor(s). To make a conductor's response causal, one must then use the *same* R_s value from equation (11) in equation (12).

In some field solvers, the inter-relationship described by equations (11) and (12) is implemented as a single surface impedance function, as shown in equation (13).

$$Z_s = R + j\omega L = \sqrt{\frac{\omega}{2\sigma}} \cdot (1 + j) \quad (13)$$

Equation (13) guarantees a causal conductor response and only requires a user to enter the conductivity for each conductor. Notice that this surface impedance function is equivalent to the frequency-varying portions of equations (11) and (12) when $R_s = \sqrt{\pi/\sigma}$.

Due to the fact that high-frequency fields and surface currents can only penetrate a very shallow distance into most conductors, the majority of full-wave field solvers cannot produce accurate modeling results by meshing and solving within the conductors. To achieve accurate results, the mesh would need to be far too dense to run in a reasonable amount of time on today's computers. To solve this problem, most field solvers use a surface impedance function, such as that in equation (13), to estimate the loss and delay caused by frequency-varying field changes within conductors. Above ~100 MHz, this method produces reasonable results.

Note that both the PCB and connector dielectric examples discussed previously use the surface impedance function of equation (13) to ensure causal output results from the models.

Summary

This paper has shown the importance of enforcing ‘base delay’ causality in passive component models in order to ensure proper time-domain wave shape. Component ‘base delay’ causality can only be achieved by implementing causal dielectric and conductor relationships within the component model itself. Early indications are that dielectric causality enforcement is more important than conductor causality enforcement. Therefore, although causal conductor relationships have been briefly outlined, a detailed method for obtaining broadband, causal dielectric permittivity data has been given. This method’s accuracy has been proven with validation data from several dielectric examples.

Ultimately, SI engineers must understand that the practice of using single dielectric constant and single loss tangent values versus frequency for a lossy model is invalid and will produce erroneous time-domain results. The use of the Σ Debye model from this paper provides a means for accurately modeling lossy, broadband components. Care must be taken, when using the Σ Debye model, to calculate the complex permittivity values at every solution frequency, rather than completing frequency-domain interpolation (as this can lead to subtle non-causalities).

In the future, a physical explanation for the Σ Debye model’s ‘aim-low’ factor, as described in the paper, would be desirable. Additionally, expanded studies on the validity of the Σ Debye model at higher frequencies and with more dielectric materials would be beneficial.

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